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13. ABSTRACT (Maximum 200 words) This report covers research performed from December 1, 1988 to June 30, 1992, in the areas of order statistics and nonparametric statistics. The principal topics investigated were (a) Median and Order Statistics Filters, (b) Comparison of Ranked Objects by Pairwise Matching, (c) Concomitants of Order Statistics in Selection Procedures, and (d) Nonparametric Analysis of Unbalanced Paired-Comparison Data or Ranked Data.					
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ORDER STATISTICS AND NONPARAMETRIC STATISTICS

FINAL REPORT

H. A. DAVID

JUNE 30, 1992

U. S. ARMY RESEARCH OFFICE

CONTRACT  
DAAL03-89-K-0010

IOWA STATE UNIVERSITY

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## 1. FOREWARD

This final report covers the period December 1, 1988 to June 30, 1992, six extra months being due to an extension granted in 1991. Six semi-annual progress reports have been issued, covering all but the last six months.

The view, opinions, and/or findings contained in this report are those of the author and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

## 2. STATEMENT OF PROBLEMS STUDIED

The PI and research assistants concentrated on the two main areas of investigation making up the title of this contract: A. Order Statistics and B. Nonparametric Statistics.

- A. Median filters are widely used in digital filtering. One simply uses the median of a moving sample of fixed size in place of the original observations in order to smooth the data. More generally, suitable linear functions of the order statistics of the moving sample may be used rather than the median. Some properties of such filters were investigated.

Concomitants of order statistics continue to be useful in selection procedures when an associated inexpensive measurement is available for the objects under consideration. An example is the selection of candidates on the basis of a screening test. If high scores are desirable, we naturally pick candidates with the highest scores on the screening test. But how many should we select for further examination or possibly further training? We study in detail a new criterion for such selection.

- B. A method for dealing with unbalanced paired-comparison data, previously proposed by the PI (Biometrika, 1987) has been examined in detail and considerably extended.

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### 3. SUMMARY OF RESEARCH FINDINGS

#### A. Order Statistics [2, 3, 4, 6, 8, 10, 11, 12, 13, 14, 15]

Research on median filters and more general order statistics filters [2] has focused on assessing performance in the presence of outliers (impulsive noise). This problem has also led to a study of the behavior of order statistics in the presence of linear trend [15].

In a Ph.D. thesis by J. Liu a detailed investigation is made of methods for comparing two groups of ranked objects by pairwise matching. The situation is illustrated by matches between two chess or tennis teams. One question is under what conditions the usual method of pairing up best with best, second with second, etc., is optimal. There is also a close connection with precedence tests, these being parallel life tests on two sets of objects, where one observes the combined ordering of failures. This research has resulted in three papers [10-12].

A major emphasis has continued to be on concomitants of order statistics, their properties and applications [3, 8, 14]. The most recent work deals with the following problem. Suppose that  $X$  is an inexpensive and  $Y$  an expensive measurement on an object or individual, also that  $X$  and  $Y$  are positively associated, and that high values are desirable. Given  $X$ -measurements for  $n$  objects, how shall we proceed to get close to  $\max(y_1, \dots, y_n)$  without actually performing all the  $n$  expensive  $Y$ -measurements? We can consider just those  $k (< n)$   $y$ 's that correspond to the  $k$  largest  $x$ 's (the concomitants of the  $k$  largest order statistics) and take their maximum. The behavior of the resulting statistic is studied in [14] with a view to helping one decide on a suitable value of  $k$ .

Visitors under the contract have co-authored or authored [5] and [6], which while falling broadly under the heading of order statistic, extend the scope of the research by the PI and his students.

#### B. Nonparametric Statistics [1, 7, 9]

In the method of paired comparisons  $n$  objects or individuals are compared pairwise by several judges. For each comparison made, a judge expresses a preference for one member, with ties sometimes permitted. The simplest situation, where each judge makes all possible comparisons, has been extensively studied. In [1] and [9] we are concerned with situations in which not all possible comparisons have been made. Our approach is nonparametric, requiring a minimum of assumptions. The same methods can be applied to incompletely ranked data by decomposing such data into its implied paired comparisons.

A method of comparing two survival distribution over a limited interval is developed in [7].

#### 4. SCIENTIFIC PERSONNEL SUPPORTED

D. M. Andrews, Research Assistant	Ph.D. (1989)
S. M. Bendre, Post-Doctoral Research Associate	
H. A. David, Principal Investigator	
S. H. Kim, Research Assistant	Ph.D. (1988)
P. Kvam, Post-Doctoral Research Associate	
* K.-D. Lee, Research Assistant	
* J. Liu, Research Assistant	
H. N. Nagaraja, Visiting Associate Professor	
J. Robison-Cox, Research Assistant	Ph.D. (1991)
C. C. Yang,	M.S. (1989)

\* Current Ph.D. students

## 5. LIST OF PUBLICATIONS

### Papers Published

1. Andrews, D. M. and David, H. A. Nonparametric Analysis of Unbalanced Paired-Comparison or Ranked Data. Journal of the American Statistical Association **85**, 1140-1146 (1990).
2. David, H. A. Some Properties of Order Statistics Filters. Special Issue. Circuits, Systems, and Signal Processing **11**, 109-114 (1992).
3. Kim, S. H. and David, H. A. On the Dependence Structure of Order Statistics and Concomitants of Order Statistics. Journal of Statistical Planning and Inference **24**, 363-368 (1990).
4. Liu, J. and David, H. A. Quantiles of Sums and Expected Values of Ordered Sums. Australian Journal of Statistics **31**, 469-474 (1989).
5. Meeden, G., Ghosh, M., Shrinivasa, C., and Vardeman, S. The Admissibility of the Kaplan-Meier and Other Maximum Likelihood Estimators in the Presence of Censoring. Annals of Statistics **17**, 1509-1531 (1989).
6. Tiago de Oliveira, J. Intrinsic Estimation of the Dependence Structure for Bivariate Extremes. Statistics and Probability Letters **8**, 213-218. (1989).
7. Yang, S. S. A Class of Nonparametric Procedures for Comparing Two Survival Distributions. Statistica Sinica **2**, 265-284 (1992).

### Papers Accepted for Publication

8. David, H. A. Concomitants of Order Statistics: Review and Recent Developments. Proceedings Volume of the Symposium on Biostatistics and Statistics in Honour of Charles W. Dunnett.
9. David, H. A. and Andrews, D. M. Nonparametric Methods of Ranking from Paired Comparisons. Proceedings of the AMS-IMS-SIAM Conference on Probability Models and Statistical Analyses for Ranking Data.
10. David, H. A. and Liu, J. Further Aspects of the Comparison of Two Groups of Ranked Objects by Matching in Pairs. Proceedings of the AMS-IMS-SIAM Conference on Stochastic Inequalities.
11. Liu, J. Precedence Probabilities and Their Applications. Submitted to Communications in Statistics - Theory and Methods.

12. Liu, J. and David, H. A. Comparing Two Groups of Ranked Objects by Pairwise Matching. Journal of Statistical Planning and Inference.

Papers Submitted for Publication

13. David, H. A. A Note on Order Statistics for Dependent Variates. Submitted to The American Statistician.
14. Nagaraja, H. N. and David, H. A. Distribution of the Maximum of Concomitants of Selected Order Statistics. Submitted to Annals of Statistics.
15. Robison-Cox, J. Tables of Order Statistics of Normal Random Variables under Linear Trend. Submitted to Selected Tables in Mathematical Statistics.